Date : 10/09/2015   Dept. No.   Max. : 100 Marks

Time : 09:00-12:00

SECTION-A

Answer all the questions   **(10x2=20 marks)**

1. Define upper integral and lower integral.

2. If f $\in$ [a,b] , g $\in$ [a,b] and if f(x)≤g(x) , then prove that $\int_a^b f \leq \int_a^b g$.

3. Evaluate : $\int_0^{\pi/2} \sin^5\theta \cos^3\theta \, d\theta$

4. Find : L[sin²t].

5. Evaluate : $\int_0^1 \int_0^2 (x^2 + y^2) \, dy \, dx$

6. If X has p.d.f f(x) = $\begin{cases} 12x^3 - 21x^2 + 10x, & 0 < x < 1 \\ 0 & otherwise \end{cases}$, find p(X ≤ ½).

7. Solve : p^2-5p+6=0.

8. Solve : $(D^2$-6D+9)y=e^{3x}.

9. State Cayley –Hamilton theorem.

10. Find the characteristic roots of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

SECTION-B

Answer any five questions   **(5x8=40 marks)**

11. State and prove Second Fundamental Theorem of calculus.

12. Show that the function f(x) = $\begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational} \end{cases}$ is not R-integrable
    on [0, 1].

13. Discuss the convergence of $\int_a^\infty \frac{1}{x^p} dx$ where a > 0.

14. Show that β(m , n) = β(n , m)

15. Find $L^{-1}\left[\frac{s+2}{(s^2+4s+5)^2}\right]$

16. Find the mgf of exponential distribution with pdf $f(x) = \lambda e^{-\lambda x}$, $\lambda > 0$.
Hence find the rth raw moment about origin.

17. Solve : $(D^2+4D+5)y = e^{2x} + \cos 4x + x^3$.

18. Examine consistency : $x+y+z=3$; $x+2y+3z=4$; $x+4y+9z=6$.

SECTION-C

Answer any two questions   **(2x10=20 marks)**

19. a) If $f \in R[a,b]$ and $a<c<b$ then prove that f   R[a,c] and f   R[c,b] and
$$\int_a^b f = \int_a^c f + \int_c^b f.$$
    b) if $f \frown R[a,b]$, show that f^2 $R[a,b]$.   **[14+6]**

20. a) Discuss the convergence / divergence of Gamma integral.
    b) State and prove the initial and final value theorems.   **[10+10]**

21. a) Solve $y'' + ay = \cos 2t$ if $y(0) = 1$, $y(\pi/2) = -1$.

    b) If (X,Y) have joint pdf $f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$

       Obtain the pdf of $U = \sqrt{x^2 + y^2}$.

                                       [8+12]

22. a) Find the eigen values and eigen vectors of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

    b) Find the inverse of the following matrix by using Cayley - Hamilton

       theorem: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ **[10+10]**

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